Physics III
ISI B.Math
Backpaper Exam : December 26,2016

Total Marks: 100
Time : 3 hours
Answer all questions

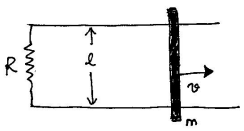
1. (**Marks:** $5+5+5+5$)

(i) Find the potential corresponding to the electric field given below. $\mathbf{E} = k[y^2\hat{\mathbf{x}}+(2xy+z^2)\hat{\mathbf{y}}+2yz\hat{\mathbf{z}}]$

(ii) A point charge is at a distance d from a grounded infinite conducting plane. How much energy is required to move the charge infinitely far from the plane ?

(iii) Consider two concentric spherical metal shells of radii r_1 and r_2. If the outer shell has a charge q and the inner shell is grounded, what is the charge on the inner shell ?

(iv) A uniform electric field E_0 in the x direction is produced by an appropriate charge configuration. A thin sheet of charge σ per unit area is placed perpendicular to the x- direction at $x = 0$. If the initial charge configuration is assumed to be undisturbed by the presence of the sheet, what is the total electric field on either side of the sheet?



2. (**Marks:** $4+5+5+6$)

A metal crossbar of mass m slides without friction on two long parallel conducting rails a distance b apart as shown in the figure. A resistor R is connected across the rails at one end ; compared with R the resistance of the bar and rails is negligible. There is a uniform magnetic field $\mathbf{B}$ perpendicular to the plane of the figure(into the paper). At time $t = 0$, the crossbar is given a velocity v_0 toward the right.

(a) If the bar moves to the right at speed v, what is the current in the resistor as a function of v ? In which direction does it flow?

(b) Find the speed v of the crossbar at time $t > 0$.

(c) How far does the crossbar go before it stops ?

(d) Show that the motion of the crossbar is consistent with the conservation of energy by showing that the energy dissipated in the resistor is exactly equal to the kinetic energy lost by the crossbar.

3. Marks (6 + 3 + 5 + 6)

(a) A small loop of wire(radius a) lies a distance z above the centre of a large loop(radius b). The planes of the two loops are parallel. and perpendicular to the common axis. Suppose current I flows through the big loop. Find the flux through the little loop. Hence find the mutual inductance.

(b) An experimental physicist claims that he has generated the following magnetic field in the laboratory by using an appropriate steady current source. $\mathbf{B} = k(xy\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + 3xz\hat{\mathbf{z}})$ where k is a constant with appropriate units. Can his result be correct ? Justify your answer.

(c) In 1897 J.J. Thomson "discovered" the electron by measuring the charge-to-mass ratio of cathode rays as follows
(i) First he passed the beam through uniform crossed electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$(mutually perpendicular and both of them perpendicular to the beam) and adjusted the electric field until he got zero deflection. What, then, was the speed of the particles (in terms of E and B)?
(ii) Then he turned off the electric field, and measured the radius of curvature R, of the beam, as deflected by the magnetic field alone. In terms of E, B and R, what is the charge-to-mass ratio $(\frac{q}{m})$ of the particles ?

4. Marks (5 + 5 + 6 + 4)

a) Write down the full set of Maxwell's equations in differential form with sources ρ and $\mathbf{J}$. What is the significance of the asymmetry between the equations involving the divergence of the electric and magnetic fields?

(b) Show that a magnetic field $\mathbf{B}$ and electric field $\mathbf{E}$ that is a solution to Maxwell's equations can always be written as

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$$

(c) An electric field which is a solution of source free Maxwell's equations is given by a monochromatic plane wave of amplitude E_0, angular frequency ω, wavelength λ and phase angle zero that is travelling in the negative x direction and polarized in the z direction. Write down the expression for the electric field and the corresponding magnetic field which is also a solution of source free Maxwell's equation. Find the energy density of the electromagnetic field corresponding to this solution.

(d) A point charge q is travelling in the along the x axis at constant speed v. Another point charge q is proceeding at the same speed along the y axis. Argue that mechanical linear momentum is not conserved for this system. No explicit calculation is required, qualitative reasoning considering the direction of the forces should suffice. Does this imply that the law of conservation of momentum does not hold for this system ? Explain.

5. Marks (8 + 5 + 7)

(a) Two point charges $3q$ and $-q$ are separated by a distance a in two different configurations (i) $-q$ at (0,0,0) and $+3q$ at $(0, 0, a)$ (ii) $-q$ at $(0, 0, -a)$ $3q$ at $(0, 0, 0)$. Find the monopole moment and the

dipole moment for each configuration. Find the approximate potential (in spherical coordinates) at large r (include both the monopole and dipole contribution).

(b) A pyramid has a square base of side a, and four faces which are equilateral triangles. A charge Q is placed on the centre of the base of the pyramid. What is the net flux of the electric field emerging from one of the triangular faces of the pyramid?

(c) Two concentric metal shells radius a and radius b ($b > a$) are separated by a material of conductivity σ. If they are maintained at a potential difference V, what current flows from one to the other ? What is the resistance between the shells ?